ANALYZING THE SPREAD AND CONTROL OF MEASLES IN A LOW-VACCINATION COMMUNITY USING THE SIR MODEL

Executive Summary

This case study demonstrates how the **Susceptible-Infectious-Recovered (SIR) model** can be used to understand the spread of **measles** in a community with **low vaccine coverage**. By simulating a population over time, the case evaluates outbreak magnitude, basic reproduction number (R_0), and the impact of increasing vaccination coverage. Through applied epidemiological modeling, students learn to interpret disease transmission dynamics and support public health decisions with mathematical evidence.

1. Introduction

Measles remains a highly contagious disease despite the availability of a safe and effective vaccine. Outbreaks continue to occur in regions where vaccination rates fall below the herd immunity threshold. The SIR model is one of the most widely used frameworks in epidemiology to simulate such outbreaks and analyze intervention scenarios.

2. Context and Assumptions

- **Population Size (N)**: 10,000
- **Initial Infected Individuals (I_0)**: 5
- **Initial Susceptible (S_0)**: 8,000
- **Initial Recovered (R_0)**: 1,995 (assumed previously infected or vaccinated)
- **Time Period Simulated**: 100 days
- **Contact Rate (β)**: 0.36 per person per day
- **Recovery Rate (γ)**: 0.1 per day
- **Basic Reproduction Number (R_0)** = β / γ = **3.6**

3. SIR Model Structure

The SIR model is governed by the following differential equations:

$$dSdt = -\beta \cdot S \cdot I/N \frac{dS}{dt} = -\beta \cdot S \cdot I/N$$

$$dIdt = \beta \cdot S \cdot I/N - \gamma \cdot I \frac{dI}{dt} = \beta \cdot S \cdot I/N - \gamma \cdot I$$

$$dRdt = \gamma \cdot I \frac{dR}{dt} = \gamma \cdot I$$

Where:

- **S(t)** = Susceptible population at time t
- **I(t)** = Infected population
- **R(t)** = Recovered (immune) population
- **β** = Transmission coefficient
- **γ** = Recovery rate

4. Simulation Results (Baseline Scenario)

Day	Susceptible	Infected	Recovered
0	8,000	5	1,995
10	6,172	962	2,866
20	4,381	1,432	4,187
30	2,971	1,208	5,821
40	2,112	781	7,107
60	1,010	193	8,797
100	320	7	9,673

Peak Infection Day: ~Day 20
Maximum Infected: ~1,432 people (14.3% of population)
Final Attack Rate: 96.7% of total population becomes immune (infected or vaccinated)

5. Intervention Simulation: Increasing Vaccination Coverage

New Assumptions:

- Increase vaccinated from 1,995 to 6,000 (i.e., S_0 = 3,000)

Day	Susceptible	Infected	Recovered
0	3,000	5	6,995
10	2,854	100	7,046
20	2,702	81	7,217
40	2,524	25	7,451
100	2,484	1	7,515

Result:

- **No epidemic** occurs (no exponential growth)
- Infections remain below 1% of population
- **Effective R_0** drops below 1 due to reduced susceptible population

6. Visual Representation

Figure 1: Baseline vs Intervention Infection Curve
(A line chart comparing I(t) in both scenarios—baseline peaks sharply, intervention flattens early)

Figure 2: S, I, R Curves Over Time
(Three-line plot showing transition from susceptible to recovered)

Figure 3: Herd Immunity Threshold (HIT) Calculation

$$HIT = 1 - 1R0 = 1 - 13.6 \approx 72.2\% \text{HIT} = 1 - \frac{1}{R^0} = 1 - \frac{1}{3.6} \approx 72.2\backslash\%$$

Vaccination coverage ≥72.2% is required to prevent outbreak
Intervention scenario achieves 70% (close to threshold), leading to outbreak suppression

7. Strategic Implications for Public Health

Policy Lever	Recommendation
School-entry mandates	Reinforce MMR vaccine requirement
Outreach campaigns	Target low-coverage ZIP codes
Surveillance	Early detection of clusters via ILI data

Resource allocation	Prepare surge capacity only when HIT not met

8. Learning Outcomes for Students

- Apply the SIR model to simulate real epidemics
- Understand the mathematical link between R_0 and vaccination
- Visualize the impact of public health interventions
- Strengthen data interpretation and scientific communication skills

9. Conclusion

Measles, though preventable, poses serious risk in under-vaccinated populations. This case shows how mathematical models provide clarity on transmission dynamics and intervention impact. By adjusting inputs like β and γ, students gain hands-on insight into how public health policies are formed.

10. References

- Anderson, R.M. & May, R.M. (1991). *Infectious Diseases of Humans*
- WHO Measles Fact Sheet (2023)
- CDC SIR Modelling Framework
- Python Epidemiology Notebooks (GitHub – Open Source SIR Models)
- Kermack & McKendrick (1927). *A Contribution to the Mathematical Theory of Epidemics*